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A CHARNOCK-BASED ESTIMATE OF INTERFACIAL RESISTANCE AND ROUGHNESS FOR INTERNAL, FULLY-DEVELOPED, STRATIFIED, TWO-PHASE HORIZONTAL FLOW

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Abstract—Charnock's (1955) relation between interfacial roughness and drag is utilized in order to determine these two quantities for fully-developed flows in tubes of arbitrary cross-section.

1. INTRODUCTION

Stratified, internal, horizontal flow has been the subject of numerous papers spanning many years and many fields of application; for a review of this subject the reader may consult such books as those by Wallis (1969), Butterworth & Hewitt (1979) and Yih (1980).

The interfacial drag and roughness are both quantities which are of intrinsic interest; they are obviously coupled, and the interfacial drag can be the dominant component of the total frictional loss. When the interfacial drag increases to sufficiently high values the flow experiences a transition to 'slug' flow (see, for example, the first two books mentioned above), and the commonly used correlation quantifying the conditions at this transition is that proposed by Wallis & Dobson (1973).

The present paper is aimed primarily at a determination of the interfacial drag and roughness, and it begins by invoking the classical relation between mean and friction velocities in fully-rough channel flow. The roughness itself is an unknown, of course, and at this stage the analysis utilises a modified form of an algebraic relation proposed by Charnock (1955) and widely used in the oceangraphy literature. Geophysical fluid-dynamics is often very different in character to typical internal flows encountered in engineering applications, but some similarities do exist, and in fact many of the papers published in the oceanographic literature refer to experiments conducted in the laboratory.

Be that as it may, it transpires that the algebraic relation obtained thereby possesses solutions only when the mean gas velocity lies below a certain maximum. It is tempting to interpret this critical condition as slug transition, and indeed it turns out that the functional dependence (at transition) of the mean gas velocity on the channel geometry, when confined to rectangular cross-sections, is identical to that of Wallis–Dobson. A direct comparison between the two correlations then fixes an unknown parameter introduced earlier into Charnock's relation; this ensures that the present theory predicts transition correctly and, moreover, quantifies the interfacial drag and roughness below transition, for general tube cross-sections.

2. THEORY

The analysis is begun by introducing the classical relation between mean and friction velocities (Goldstein 1965, Schlichting 1960) for fully-rough internal flow:

$$u_G/u_\tau = 5.75\log_{10}(R/\epsilon) + 4.73 \qquad [2.1]$$

u_G, u_τ are the mean and friction velocities, R is the mean hydraulic radius $2A/C$ (A and C are the cross-sectional area and perimeter respectively), and ϵ is the RMS roughness scale (which is normally uniformly distributed around the perimeter).

Charnock (1955) employed dimensional arguments, subsequently supported by theoretical considerations (Phillips 1977), to propose that $\epsilon \propto u_\tau^2/g$, where g is the gravitational acceleration. Numerous laboratory and field experiments, published in the oceanography literature (e.g. Wu 1968 and Phillips 1977), have indicated that:

$$\epsilon \simeq 0.33\, u_\tau^2/g \qquad [2.2]$$

provided $u_\tau \geq 0.5\ \mathrm{m\ sec^{-1}}$. There is some disagreement in the literature about the value of the constant of proportionality in [2.2], and Hsu (1974) has explained the discrepancies in terms of variations of mean wave slopes from experiment to experiment. However, there is no need to be precise at this stage of the analysis because of the introduction of a disposable constant below.

It should be mentioned that the role of the gravitational acceleration g is expected to be represented by the buoyancy term $(\rho_L - \rho_G)g$, and since the mean interfacial stress is $\rho_G u_\tau^2$, one may anticipate that $\epsilon \propto \rho_G u_\tau^2/g(\rho_L - \rho_G)$.

A similar approach is followed in this paper. However, it is necessary to account for the fact that the roughness is not uniformly distributed around the channel gas-phase perimeter and that the interface between the liquid and the gas is mobile, so Charnock's relation is modified to:

$$\epsilon_e = 0.33\xi u_\tau^2/g;\ \ \xi = \eta l/c \qquad [2.3]$$

where ϵ_e is an "effective" roughness, l is the length of the interface in the cross-sectional plane (see figure 1), and η is a constant yet to be determined.

Substitution of [2.3] in [2.1] then yields an algebraic relation between u_τ and u_G:

$$\frac{u_G}{u_\tau} = 5.75 \log_{10}\left(\frac{20.1\ V^2}{\xi u_\tau^2}\right);\quad V^2 = gR \qquad [2.4]$$

What is of particular interest is the fact that, for a given value of ξ, there is a maximum value of u_G, denoted by $(u_G)_T$, beyond which no solutions of this algebraic equation exist (see figure 2). When $u_G < (u_G)_T$ two solutions exist, identified by A and B in the figure; however, since the stress should increase with u_G, point B is physically meaningless, and A is the relevant solution. As u_G is increased and reaches the value $(u_G)_T$, the two points coalesce, and the two curves representing the left and right-hand sides of [2.4] are tangential at point C. Alternatively, if [2.4] is regarded as an expression of u_G as a function of u_τ, then the transition corresponds to

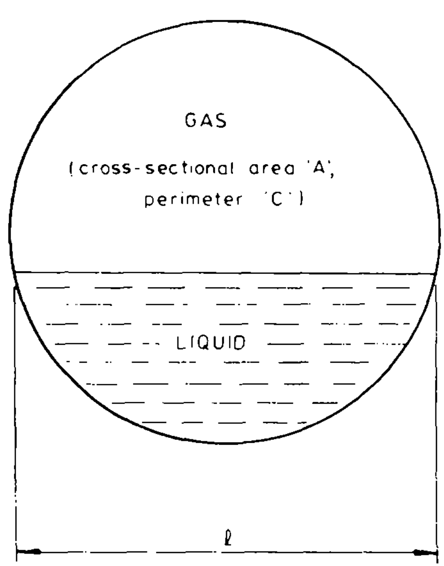

Figure 1. A typical cross-section.

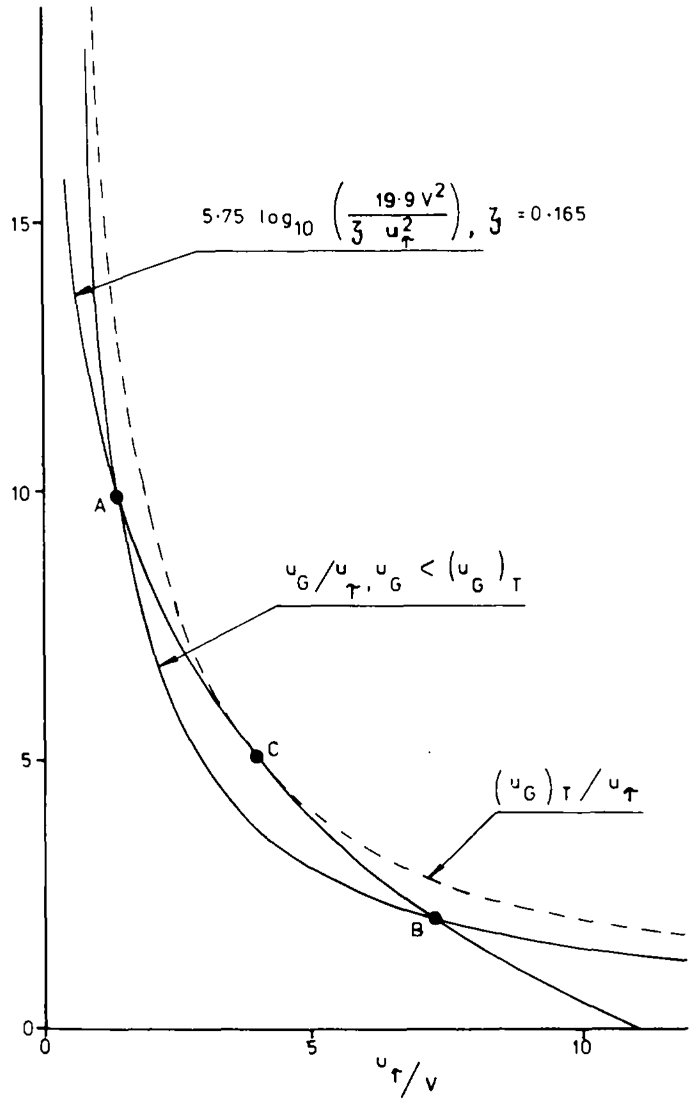

Figure 2. A sketch of the terms in [2.4], as functions of the friction velocity.

$\mathrm{d}u_G/\mathrm{d}u_\tau = 0$. By differentiating [2.4] with respect to u_τ and substituting back into that equation, one immediately finds that:

$$(u_G/u_\tau)_T = 5.0 \qquad [2.5a]$$

$$\left(\frac{u_\tau}{V}\right)_T = \frac{1.64}{\xi^{1/2}}, \quad \left(\frac{u_G}{V}\right)_T = \frac{8.25}{\xi^{1/2}}. \qquad [2.5b]$$

Generally,

$$\frac{V}{\xi^{1/2}} = \left(\frac{2Ag}{\eta l}\right)^{1/2}. \qquad [2.6]$$

This expression is particularised to a flow in a rectangular channel in order to perform the comparison with the Wallis–Dobson stratified-slug transition correlation (Wallis & Dobson 1973, Butterworth & Hewitt 1977):

$$\frac{V}{\xi^{1/2}} = \left(\frac{2gh}{\eta}\right)^{1/2} \quad \text{(rectangular channel).} \qquad [2.7]$$

Here h is the height of the gas-phase channel. Now the Wallis-Dobson correlation is, in the present notation,

$$(u_G)_T = \frac{1}{2}\left[\left(\frac{\rho_L - \rho_G}{\rho_G}\right)gh\right]^{1/2} \qquad [2.8]$$

where ρ_L and ρ_G are the liquid and gas densities respectively. Comparing [2.5] and [2.8] then leads to:

$$\eta \simeq \frac{545\rho_G}{\rho_L - \rho_G}. \qquad [2.9]$$

For air flowing over water under atmospheric conditions, $\eta \simeq 0.67$, and for the purpose of illustration these results are presented in figure 3 as curves of u_τ/V vs u_G/V, with l/C having the values 0.1, 0.25 and 0.45. The dotted line $u_\tau = 0.2u_G$ represents the transition locus, at which the curves acquire infinite slope.

The results can be collapsed onto a single universal curve by employing the following coordinates:

$$\hat{u}_G = \left(\frac{\xi}{20.1}\right)^{1/2}\frac{u_G}{V}, \quad \hat{u}_\tau = \left(\frac{\xi}{20.1}\right)^{1/2}\frac{u_\tau}{V} \qquad [2.10]$$

In terms of these variables, [2.4] reads

$$\hat{u}_G = -5\hat{u}_\tau \ln \hat{u}_\tau \qquad [2.11]$$

and the stratified-slug transition corresponds to

$$u_{\tau_T} = e^{-1} \simeq 0.368, \quad \hat{u}_{G_T} > 5\,e^{-1} \simeq 1.839. \qquad [2.12]$$

Parts of the analysis have, admittedly, been biased towards rectangular ducts, but the following points should be borne in mind: (i) [2.4], [2.5], [2.11] and [2.12] are quite general and are expected to apply to conduits of arbitrary cross-section. (ii) In this paper it is suggested that [2.9] may be applied to non-rectangular ducts. (iii) If [2.9] is found to fail for a particular geometry, it can simply be modified in precisely the manner utilized here, by comparing [2.5b] with experimental slug-transition data. In principle, it is possible to determine η with measurements at a single void fraction, since the channel geometry is embodied in the definition of ξ and in [2.6].

Equation [2.11] has been plotted in figure 4, together with the curve

$$\hat{u}_\tau/\hat{u}_{\tau_T} = 1 - [1 - (\hat{u}_G/\hat{u}_{G_T})^2]^{1/2} \qquad [2.13]$$

which is a more convenient, explicit relation offering reasonable accuracy at conditions not far removed from transition.

In terms of the customary skin-friction coefficient, $C_f = 2(u_\tau/u_G)^2$, if follows from [2.5a] that at transition $C_f = (C_f)_T = 0.08$. It should be emphasized that the present analysis only accounts for the contribution of the interface to the frictional losses, although that contribution is often the dominant one. There is an interesting similarity between figure 3 and figure 5 presented by

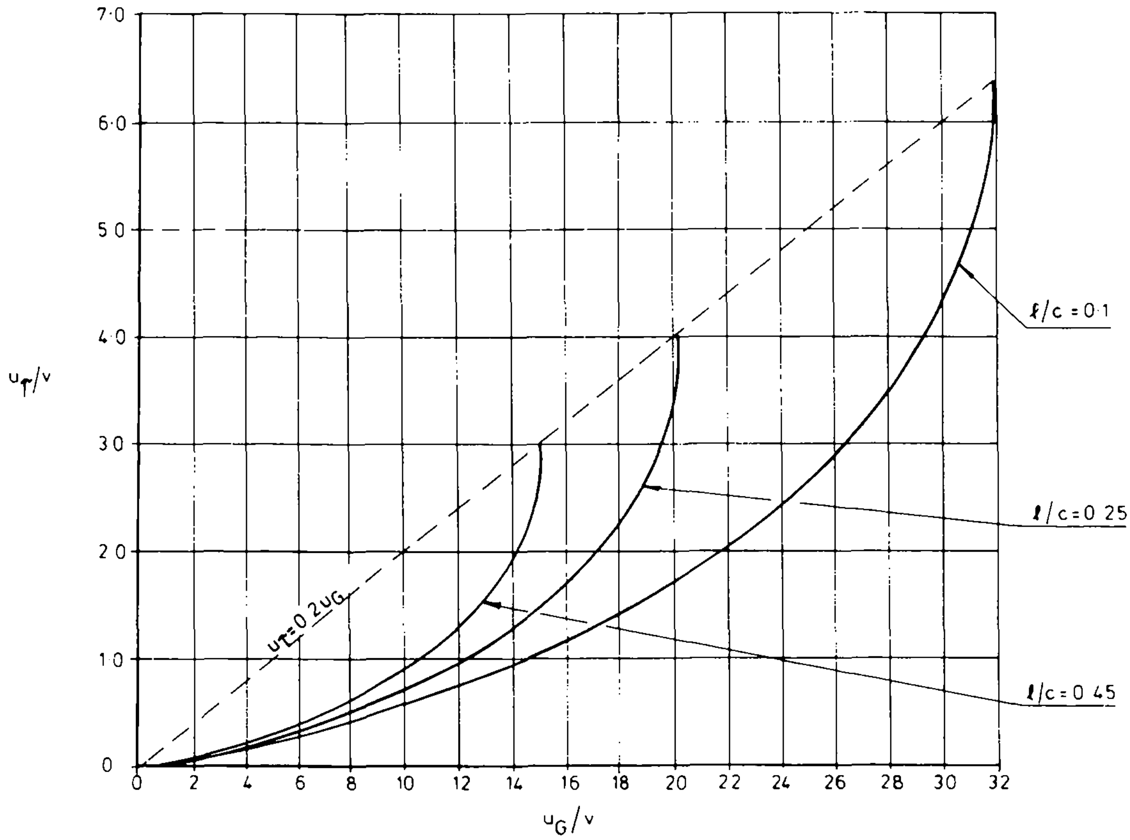

Figure 3. Friction velocity versus mean gas velocity, $\alpha = 0.66$.

Wallis & Dobson (1973) in which the gas velocity was plotted against channel slope needed to maintain uniform liquid depth (the channel slope was proportional to the interfacial drag). Unfortunately, it is not possible to execute a direct comparison between the current predictions and Wallis and Dobson's results, because they did not publish sufficient information. This may be seen from the following argument: Considering planar, fully-developed, stratified flow, the momentum equation may be integrated once to describe the customary linear shear stress distributions in the two fluids and these in turn yield the following expressions for the stresses at the floor and the roof of the channel respectively:

$$\tau_B = \tau_I - h_L\left(\frac{dp}{dx} + \rho_L g \sin\theta\right) \quad [2.14a]$$

$$\tau_T = \tau_I + h_G\left(\frac{dp}{dx} + \rho_G g \sin\theta\right) \quad [2.14b]$$

Here τ_I is the interfacial stress, h is the depth of the respective fluid, dp/dx is the pressure gradient, and θ is the channel's angle of inclination to the horizontal. τ_I may be inferred in an experiment if dp/dx and either τ_B or τ_T are measured. Alternatively, dp/dx may be eliminated to relate τ_I to τ_B and τ_T exclusively:

$$(h_L^{-1} + h_G^{-1})\tau_I = (\rho_L - \rho_G)g \sin\theta + h_L^{-1}\tau_B + h_G^{-1}\tau_T. \quad [2.14c]$$

There is little point in employing one of the empirical correlations for pressure drops because

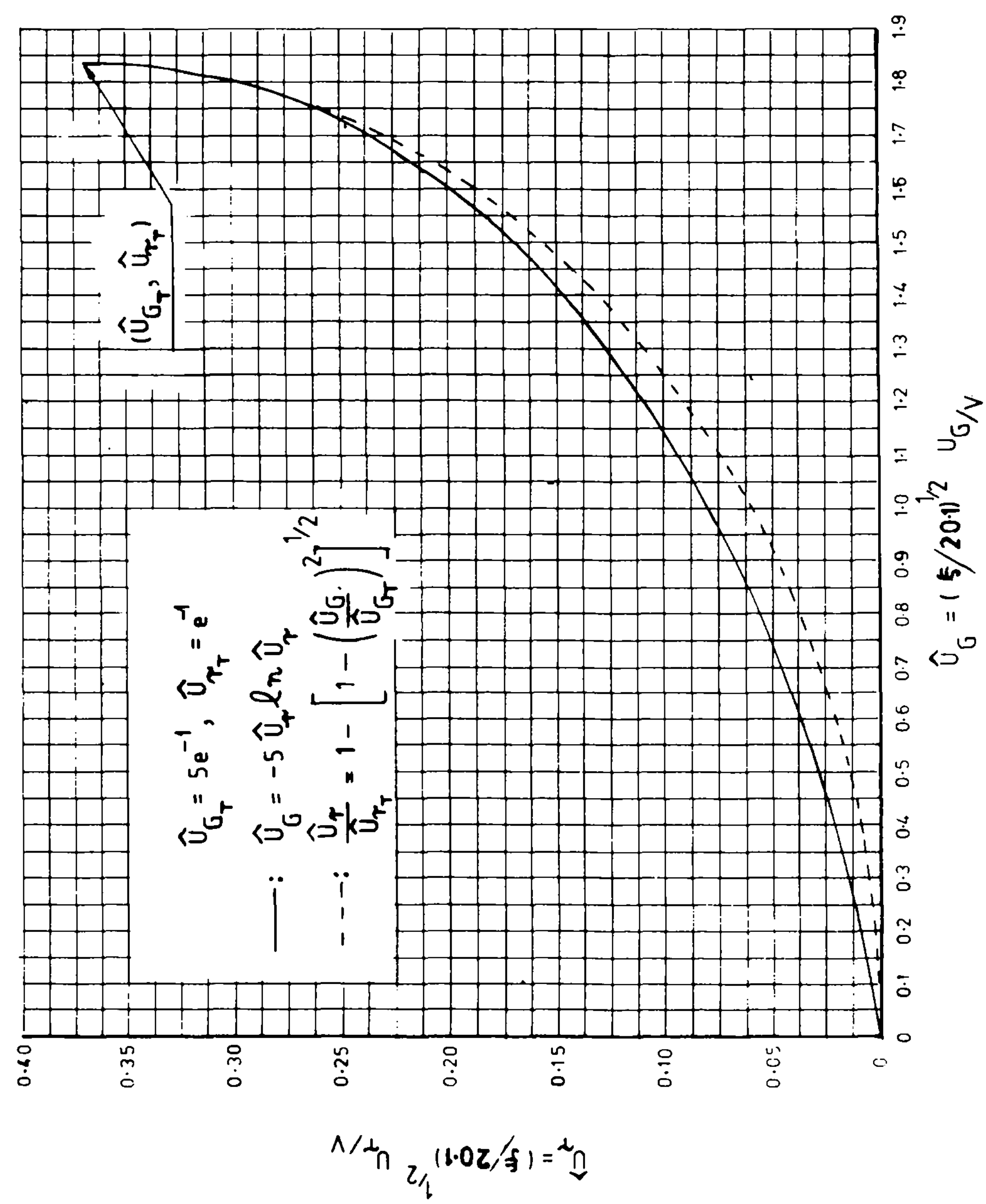

Figure 4. Universal stress curve.

the inherent errors would only serve to cast doubt on the validation, and experimental tests of the present theory must await the appearance of the additional quantities delineated above.

Expressions [2.9] diverges as $\rho_G \to \rho_L$, and the present theory will probably break down under such circumstances because gravitational influences will become inappropriate vis-à-vis Charnock's relation.

As it stands, the analysis assumes zero liquid flow, and non-zero liquid flows are considered by replacing u_G by $u_G - u_L$; the reader is referred to the pertinent comments by Wallis & Dobson (1973).

CONCLUSIONS

Charnock's (1955) relationship, modified to account for boundary effects in internal flows, has been used in conjunction with the classical expression for the friction in fully-rough channels in order to predict the interfacial shear stress and roughness under fully-developed conditions.

A mathematical "breakdown" has been shown to correspond to the Wallis–Dobson (1973) slug transition correlation, with the aid of which an unknown parameter, introduced earlier in the analysis, is determined in terms of the two fluid densities. The interfacial friction coefficient is predicted to have a value of about 0.08 at transition.

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